

Stochastic Modeling of the Stray Capacitance of Air-core Wound Inductors

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Abstract—In this work, the stochastic capacitive effect in a multilayer air-core inductor is investigated using two deterministic models: one based on the finite element method and the other on an analytical approach. Both are compared through two geometric uncertainties models and show a good agreement. An experimental validation will also be carried out on a batch of wound inductors in order to impose a reference for computed stray capacitances.

Index Terms—Stray capacitance, finite element method, analytical method, geometric uncertainties, air-core inductors

I. INTRODUCTION

Stochastic models are based on the representation of random input vector components and can become cumbersome as the dimension increases. Such models are essential when designing robust components capable of withstanding fluctuations in physical and environmental parameters. This is the case for instance with parasitic capacitances involved in the characterization of multilayer wound inductors, particularly when considering the geometric uncertainties associated with the positions of conductors in the winding window as addressed in this contribution. Considering a wide range of frequency, such a characterization can be computationally intensive using numerical method, such as, e.g., finite element (FE) analysis. Hence, another approach, based on a less demanding analytical method, may prove to be suitable to this type of study.

Stray capacitances in air-core wound inductor, can be determined through the electrostatic energy stored in the dielectric medium and from the fact that the conservation of energy holds equally for lumped circuits and electromagnetic fields. And this is strongly linked to the positions of conductors in the winding window. Since these positions are random variables, the bulk capacitance also becomes random. To our knowledge, few studies have addressed the impacts of geometric uncertainties associated with positions of conductors in the winding window more specifically evaluation on parasitic capacitances in multilayer wound inductors. In [1], the authors consider two winding patterns (orthogonal and hexagonal) as extreme cases bounding geometric uncertainties in a stochastic study associated with stray capacitance in wound air-core inductors. That paper considers uniform distributions of pitch

and permittivity parameters to predict the range of variability of stray capacitances.

In this work, we focus on the propagation of geometric uncertainties (related to the positions of the conductors in the winding window) for the extraction of the bulk stray capacitance of multilayer wound air-core inductors. The proposed stochastic analysis is based upon a deterministic, analytical or numerical model, included in a Monte Carlo simulation. In addition, two approaches for incorporating the geometric uncertainties associated to the positions of conductors are implemented: one that emulates the manual winding process [2] (hereafter referred as method 1) and another in which the conductors are randomly deployed in the winding window around a given pattern (referred to as method 2). Note that a uniform distribution is assumed in that case. The remainder of this proposal is organized as follows. First, the overall methodology is described in Section II. Then, the simulation results and comparison are presented in Section III. Finally, conclusions are drawn in Section IV highlighting future work for the final paper.

II. METHODOLOGY

The adopted methodology is based on the propagation of geometric uncertainties in a deterministic model. Numerical (based on a 2D electrostatic formulation [2], [3]) and analytical models are considered in this work. Both models assume a linear distribution of the potentials along the winding turns in each layer. For the analytical model, the hypothesis of straight electric field lines between conductors is also adopted (see Fig. 1). Although it is less accurate than FE analysis, the advantage of this model lies in its low computational power demand. This deterministic analytical model includes turn-to-turn and layer-to-layer capacitances [1]. Both depend on the length of electric field line $x(\theta)$ between adjacent conductors according to the equation:

$$C = \int_{\theta_1}^{\theta_2} \frac{\varepsilon_0 l_x}{2 \left(\frac{x(\theta)}{d_0} + \frac{1}{\varepsilon_{ins}} \ln \frac{d_0}{d_c} \right)} d\theta \quad (1)$$

where $x(\theta) = p - d_0 \cos \theta$ involves the distance p between adjacent conductors; and d_c and d_0 are the conductor diameter and conductor+insulation diameter, respectively. Moreover, l_x

represents the mean length of turns involved in the calculated capacitance; ε_{ins} is the relative permittivity of the insulation, and $\varepsilon_0 = 8.85$ pF/m, the permittivity of the air.

The total stray capacitance is obtained analytically using the following equation:

$$C_{tot} = \frac{1}{(N-1)^2} \left[\sum_{k=1}^{N_l} \sum_{i=1}^{N_m-1} C_{tt}(k, i) + \sum_{k=1}^{N_l-1} \sum_{i=1}^{2N_m-1} i^2 C_{ll}(k, i) \right] \quad (2)$$

where C_{tt} and C_{ll} represent turn-to-turn and layer-to-layer capacitances. These are calculated by using (1). N is the total number of the turns.

Since Monte Carlo simulation is a non-intrusive method, it is adopted in this work for the propagation of geometric uncertainties into each deterministic model (analytical or FE). Finally, we end up with four stochastic models each resulting from the combination of one of the two deterministic models and one of the two above-mentioned methods for modeling the geometric uncertainties associated with the positions of the conductors in the winding window.

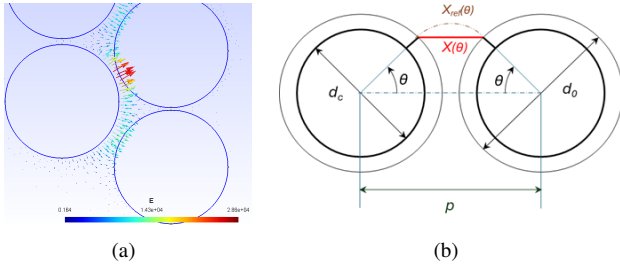


Fig. 1. Illustrating (a) electric field between adjacent conductors and (b) details on the parameters involved in the stray capacitance formula.

III. SIMULATION RESULTS

Simulations are carried out on a 44-turn axisymmetric air-core inductor distributed over 4 layers, each containing 12 turns per layer except for the last layer; $d_c = 0.37$ mm, $d_0 = 0.375$. The bobbin diameter equals 3.7 mm. The conductor insulation is characterized by $\varepsilon_{ins} = 4.2$ and the winding pattern is hexagonal. Due to the high dimension of the random input vector for incorporating the geometric uncertainties according to model 2, a large number (a few thousand here) of Monte Carlo iterations will be required for better convergence [2]. Yet, the CPU time is significantly reduced thanks to the lightweight deterministic model. The distributions of the bulk stray capacitance obtained by the analytical and numerical (FE) models are shown in Fig. 2. In each case, the results obtained by using both methods for incorporating geometric uncertainties are compared. A large dispersion associated with a large variability of the random input vector is observed for the geometric uncertainties included in the stochastic model according to method 2.

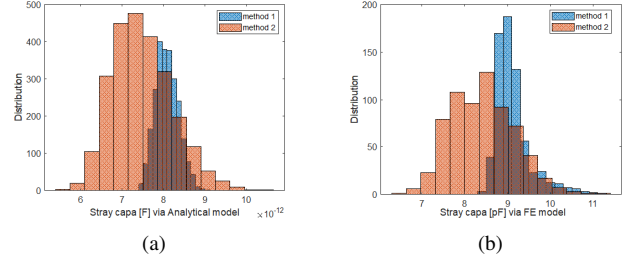


Fig. 2. Stray capacitance distributions resulting from (a) analytical and (b) FE model (geometric uncertainties included according to methods 1 and 2).

Table I presents the comparison between the four stochastic models. In terms of CPU time, the two stochastic models based on the analytical computation of the total stray capacitance are advantageous in terms of execution time (less than one minute for 2500 MC iterations). One can notice a good agreement between different models with a difference of 0.5 pF (in terms of mean value) between methods 1 and 2. It is worth noting also that method 1 requires a reduced number of iterations for the convergence (perceived here as small fluctuations in the adapted mean) of Monte Carlo simulation.

TABLE I
RESULTS COMPARISON ON COMPUTED STRAY CAPACITANCES FROM FOUR STOCHASTIC MODELS.

Stray capa. [pF] + CPU time [s]	FE model ^a	Analytical model ^b
Geometric uncert. (method 1)	9.12 (783 s)	8.08 (42.2 s)
Geometric uncert. (method 2)	8.65 (884 s)	7.56 (50.6 s)

For ^a676 and ^b2500 Monte Carlo iterations.

IV. CONCLUSION

In this work, four stochastic models for evaluating the capacitance of wound air-core inductors were compared in terms of prediction and computational speed. Models based on analytical calculation of the total stray capacitance presented interesting advantages in terms of computational speed independent of the dimensionality of the random input vector. This dimensionality depends on the method adopted for incorporating the geometric uncertainties. In the full paper, an experimental validation will be carried out on a batch of air-core inductors via an impedance analyzer and taking care to limit the frequency up to which the assumption of potential linearity remains valid.

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